Non-Linear Analysis of Bolted Extended End-Plate Steel Beam-To Column Connection Damage Detection for Slab using Different Model Updating Algorithmns

Fatin Nadiah Abdul Rahman, Norhisham Bakhary

Faculty of Civil Engineering, Universiti Teknologi Malaysia, Malaysia

norhisham@utm.my

Keywords: Structural health monitoring; vibration-based damage detection; algorithm.

Abstract. Vibration-based damage detection (VBDD) is one of the techniques used in Structural Health Monitoring (SHM) for damage detection in structure. VBDD provides more efficient, reliable and economical method to improve the safety and reduce the cost of maintenance in structural engineering. This paper investigates the performance of model updating (MU) method using different type of algorithms in damage detection. A finite element analysis is performed to obtain dynamic properties of undamaged and damaged structure to be trained in model updating process using 3 different types of algorithms. Three optimization functions of different algorithms used in this study are constrained optimization (fmincon), least-square optimization (lsqnonlin) and multiobjective optimization (fgoalattain). Different damage cases are introduced at different location with different damaged intensities by reducing the modulus of elasticity (E) value for corresponding segment. The performance of MU is evaluated by Stiffness Reduction Factor (SRF). The comparison of SRF for actual value and predicts value is made to check the applicability of three types of algorithms in damage detection. Mean Square Error (MSE) is applied to evaluate stopping and tolerance criteria effect on result improvement. The results show all three types of algorithms capable to provide reliable results in damage prediction and an improvement of result has been made when increased the stopping and tolerance criteria

Introduction

Recently, researchers have developed various types of techniques in order to produce a reliable approached to improve the safety and reduce the cost of maintenance in structural engineering. According to [1] SHM is a process of implementing a damage detection strategy. This process will provide the information about the capability of the structure to continue to perform its desired function in the future due to aging and degradation caused by environmental impact. VBDD is one of the techniques used in structural health monitoring. It applied the concept of degradation of structural design would result the change in dynamic properties such as natural frequencies, mode shapes and damping where the degradation of the structure is represent by changes in material and geometric properties. Numerous studies have been conducted to verify the abilities of dynamic properties in detecting and locating the damaged.

In this study, model updating is performed using sensitivity (iterative) method to minimize the discrepancies between two results for damaged detection. Model updating using sensitivity method is utilized in this study because it is capable to provide more reliable result according to previous sensitivity studies conducted. Finite element model (FEM) is required to generate the modal data of structure to be used in model updating process. According to [2], model updating is basically a process of adjusting certain parameters of the FEM until an objective function is met. The selection of updating parameter and the construction of the objective function is the critical part in model updating in order to provide better convergence. The process of model updating is repetitive until the differences between the objective function become smaller than consecutive iteration. Basically the damaged detection is nonlinear function of selected updating parameters and the inverse process that need the application of algorithms for evaluation. In this study the performance of different algorithms using model updating in damaged detection is performed using *fmincon, lsqnonlin* and *fgoalattain* optimization function.

Previous Studies

VBDD is a non-destructive evaluation of structural analysis which offers more efficient and reliable method to detect damage at the earliest stage. Using experimental method, damage detection only can be evaluated near to the surface area of the structure and require the vicinity of the damage known as a priori. The drawback of the experimental method can be improved by using VBDD method. Nowadays, VBDD has been widely used in many engineering sector such as civil, mechanical and aerospace to examining changes in structure on the modal parameters. According to [1], in earlier study of VBDD has been started in oil industry to examine the damage in offshore platform. Study in [7], stated that there are many factors that contribute to the increasing of vibration-based damage identification method application. The main factor is the failures of structures and mechanical systems used by the public. The failure of in-flight loss of the exterior skin on an Aloha Airlines in Hawaii and the collapse of the Tacoma Narrows Bridge in Washington have gained public's concern and influenced the development of this method. Furthermore, advance in technology especially in computing and the adaptation and advancement of the finite element method contribute to the rapid development of vibration based damage detection.

The basic concept of VBDD is any damage occurs in physical properties of the structure will promote the changes of modal properties. The physical properties that can influence the modal properties (natural frequencies, mode shapes and modal damping) are mass, stiffness and damping. The equation of motion in Eq. 1 shows the relationship of physical properties modal properties that been utilized in VBDD.

$$M\ddot{x} + C\dot{x} + Kx = 0 \qquad (1)$$

where M is the mass matrix, C is the damping matrix and K is the stiffness matrix. $\ddot{x}$, $\dot{x}$ and x are vectors represent acceleration, velocity and displacement respectively. The associated eigenvalue problem shows in Eq. 2.2 as follows:

$$(\omega_{i^2} M + j\omega_{i^2} C + K)\, \phi_i = 0 \qquad (2)$$

where ω_{i^2} and ϕ_i are the i^{th}modal circular frequency and mode shape and $j$is the imaginary unit. The damage cannot be express if there are no significant differentiations can be made between two different states of the system. Damage according to [1] is defined as the change of material and/or geometric properties of structural system including the changes to the boundary condition and system which influenced the performance of the system.

There are three type of data utilized in vibration-based damage detection which can be classified as time domain, frequency domain and modal domain. In time domain, the respond can be measured as time history using displacement, velocity and acceleration from the vibration test using sensors positioned in the structure. The ARMA model and ARMAX model are vibrating systems used to analysis time series. [9] stated that the problem of structural dynamics is difficult to analyse using time domain due to high number of degree of freedom. Hence it required appropriate reduction method of data. The numerous volumes of data from time domain can be reduced and converted by using Fast Fourier Transform algorithm to Frequency Respond Data (FRF) which can be classified as frequency domain. In review by [7] stated that in further analysis, the extraction of frequency data domain is carried out to produce modal quantities namely natural frequencies, mode shape and modal damping value as modal domain data. Besides, many literatures has concentrated the method of using modal domain due to (i) the early literature concentrate on modal domain and (ii) natural frequencies and mode shape can be provide more attractive than abstract characteristics from frequency domain and time domain. The analysis of modal domain also easier to interpret due to smaller amount of data produced. [8] has categorized damage identification method based on four levels of damage which are :

1. Level 1 : Determination that damage is present in structure
2. Level 2 : Level 1 plus determination of the geometric location of the damage
3. Level 3 : Level 2 plus qualification of severity of the damage
4. Level 4 : Level 3 plus prediction of the remaining service life of structure

Level 1 and Level 2 damage identification do not use any structural model primarily. Level 3 predictions can be achieved in some cases when vibration-based methods combined with structural model. According to [1], Level 4 prediction basically related with the fatigue-life analysis, fracture mechanic discipline and structural design assessment.

Numerous and continuous studies have been conduct to improve the performance of VBDD in damage detection. A large number of techniques, methods and algorithms have been developed to overcome the problems arise in analysing the complex and also basic structural system. Several methods used in VBDD methods are:

1. Damage Detection Based On Changes in Basic Modal Properties
2. Mode Shape Curvature Changes
3. Model Updating Based Method

Damage detection based on changes in basic modal properties essentially using basic modal properties comprise of resonant frequencies, modal damping and mode shape vector. By using frequency change the application for damage identification have significant practical limitation due to low sensitivity to damage as it need large level of damage in order to get precise measurement [6]. Recently, studies shows that resonant frequency has much less statistical variation from random error compared to other modal parameters. However, changes of modal frequencies in damage detection provide an insufficient data to determine the location of the damage uniquely [1].

Using mode-shape vectors in damage detection can examine the differences between the measured modal vector of damaged states and undamaged states. The information provides by mode shape vector had the ability to locate the damage as they are local parameter of structural system that has high sensitivity in small changes of the system. The use of MAC and scaled mode shape changes is pointed out to have better identification of damaged location [6].

The concept of mode shape curvature changes method is damaged is defined by the change of curvature from derivation of mode shape. This derivation is obtained using second derivative of mode shape vector that is more sensitive to small perturbation in system [1]. Hence, there is a direct relationship between curvature and bending strain for beam, plate and shell. According to [6], changes in curvature can provide damage identification result for FEM beam structure and the curvature is computed from the displacement mode shape using the central difference operator. However, reviewed by [6] discovered that calculating curvature from mode shape numerically resulted in unacceptable error and the improvement of the result can be obtained using measured strain instead of curvature.

Another class in VBDD is model updating-based method. This method is based on the concept of comparison between modal parameter obtained from numerical model of FE modelling and experimental model from vibration tests. The model updating is a process of adjusting certain parameters of finite element model until it closely matched with the measured modal data. Model updating can be classified into 2 categories which are direct method (non-iterative) and sensitivity method (iterative). Direct method is the computational of mass matrix and stiffness matrix using closed-form direct solution. The problem is generally formulated as a Lagrange multiplier or penalty-based optimization [6]. Study by [11] conclude that the detailed correlation procedure are required to establish a set of reliable test data as postprocessor to verify the accuracy improvement and physical validation of the updated model.

Sensitivity (iterative) method is the most popular method and widely use in damage detection due to the capabilities in solving inverse problems. According to [10], the more precise and completeness in measured data and accuracy in numerical model can be achieve through (i) higher modes of numerical matrix system presented by stiffness matrix provide realistic stiffness model and (ii) the parameters of numerical model are related directly to geometry and material properties of structure may provide a meaningful model which direct method are not capable to.

The procedure of iterative method can be obtained in [12]. Selection of parameters to minimise the objective function has becoming the most crucial part in optimization to ensure the effectiveness of model updating process. A sensitive design parameter is required in model updating to compute the correlation between two different type of system which are from analytical data and measured data in objective function. Study by [5] conclude that the selection of geometrical and material properties as updating parameters provide better result rather than choosing geometrical alone. The updated design parameters are then iteratively adjusted until it the different between objective function become smaller. The iteration stops when the objective function is achieved.

The main difference between various sensitivity-based approaches is the selection of the residual in objective function. Performance of model updating process greatly depends on the selection of residual in the objective function [13]. Study in [14], using MAC to evaluate the correlation between the experimental and analytical mode shape in objective function and the utilization of elastic modulus, E and moment of inertia, I_y as updating parameters provide a good result in damaged detection. While study by [15] prove that the objective function considering 11 measured frequencies with 10 updating structural parameters also can produce reliable result in damage detection of 1/150 scaled suspension bridge model.

Besides, study by [4] considered the measure of modal frequency only, mode shapes only and both modal frequency and mode shape in objective function. The elastic modulus E is used as updating parameters and the updating is performed using the multiobjective optimisation function (*fgoalattain*) in Matlab. The results show that the consideration of modal frequency only is not sufficient to detect damage in symmetrical structure. However consideration of both modal frequency and mode shape can detect damage location and severities successfully with minimum error compare to consideration of mode shapes only.

The different between various sensitivity-based FE model updating algorithms can be found in (i) objective function, (ii) numerical approach used to implement the optimization and (iii) algorithms to obtain the sensitivity matrix [16]. Nonlinear least squares minimization in frequency domain has been conducted in [17]. The applicability of nonlinear least squares minimization with the gauss-Newton algorithms provide reliable result as location, severities and speed of iteration in damaged detection is successfully achieved.

The comparative study on different algorithms in [18] is between the global optimization (Couple Local Minimizer) and local optimization (Levenberg-Marquardt algorithm, Sequential Quadratic Programming and Gauss-Newton method). In global optimization, the local optimization is coupled so that better solution is obtained. The performance is improved as it can accurately detect, localize and quantify the damaged although in the present of noise. Even though, Levenberg-Marquardt algorithm, Sequential Quadratic Programming and Gauss-Newton method are capable to detect the location and severities of the damaged, the method of coupling these local optimization able to provide reliable result with high accuracy.

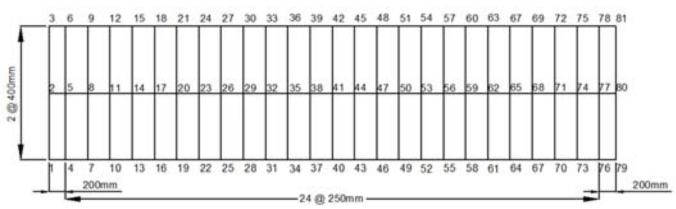
Table 1: Drawbacks of VBDD methods

Method	**Drawbacks**
Damage Detection Based On Changes in Basic Modal Properties	• need a large number of measurements so that the mode-shape vectors can accurately characterize • insufficient number of frequencies lead to poorly locating of the damage • noise contamination
Mode Shape Curvature Changes	• calculating curvature from mode shape numerically resulted in unacceptable error
Model Updating Based Method	• ill-conditioning optimization in inverse problem evaluation • high time consuming and higher number of iteration due to higher DOF

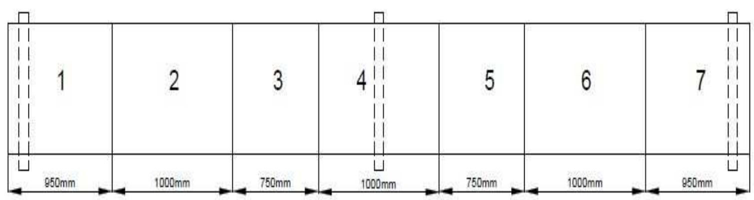
Several method in VBDD are developed to overcome problem arise due to surrounding and complexity of structural system. Table 1 below shows the drawback of different VBDD methods.

Methodology

There are 2 major parts in this study: i) Development of RC slab numerical model using FEM for undamaged structure and damaged structure and ii) model updating process using (*fmincon*), (*lsqnonlin*) and (*fgoalattain*) functions. Mode shapes data generated from part i) is utilized in ii) for damage detection. In part i) a two-spanned simply supported reinforced concrete (RC) slab with dimension 6400mm x 800mm x 100mm is generated. The length of each span is 3000mm and 200mm overhang at both ends. Material properties used are young modulus, E = $3.3\text{x}10^{10}$ N/mm^2, density, ρ = $2.45\text{x}10^3$ kg/mm^3 and Poisson's ratio, v = 0.2. The slab is considered as shell elements with 81 nodes and 52 elements and divided into 7 symmetrical segments. Figure 1 (i) and (ii) shows the dimension of the slab. Three damage cases are introduced at different location with different damaged intensities by reducing the modulus of elasticity (E) value for corresponding segment as shown in Table 2.



i) Finite element mesh with node number



ii) Slab segment

Figure 1: RC slab dimension

Table 2: Stiffness, E values for Case 1, Case 2, and Case 3

Segment	1	2	3	4	5	6	7
Undamaged	E	E	E	E	E	E	E
Case 1	E	0.7E	E	E	E	E	E
Case 2	E	E	E	E	E	0.5E	E
Case 3	E	E	0.7E	E	E	0.7E	E

Table 3: First three frequencies of the slab in different damaged

Mode	Undamaged	Case 1	Case 2	Case 3
1^{st} mode	18.636	17.571	16.399	17.057
2^{nd} mode	29.355	28.185	27.314	27.985
3^{rd} mode	76.213	74.233	71.868	71.813

Due to difficulties in obtaining higher modes in real practice only the first three mode shapes are considered. The mode shapes are measured on the centreline along the span. Figure 2 illustrates the mode shapes of undamaged slab for first three modes. The same steps are repeated for Case 1, Case 2 and Case 3.

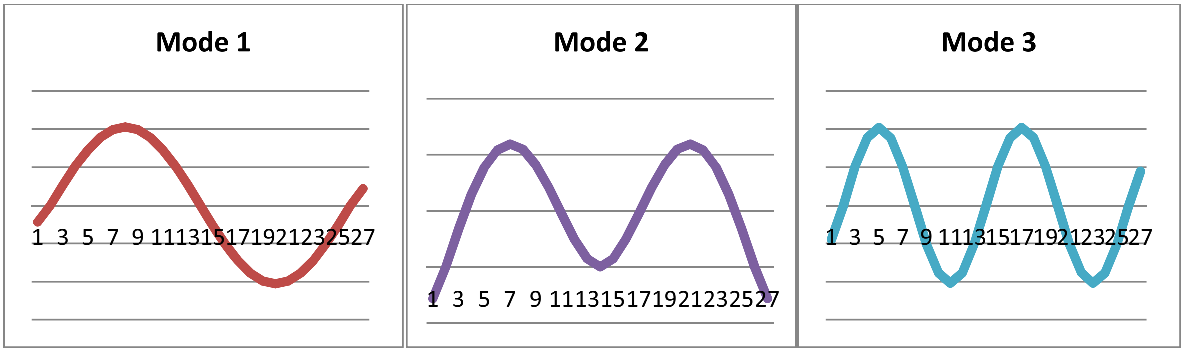


Figure 2: First three mode shapes of undamaged slab

In part ii) E value of each segment is used as the updating parameters and mode shapes generated from part i) is employed in Modal Assurance Criterion (MAC) evaluation as objective function using different algorithms. The *fmincon* optimization, *lsqnonlin* optimization and *fgoalattain* optimization function used in this study are shown in Eq. 3.1, Eq. 3.2 and Eq. 3.3 respectively and Eq. 3.4 is the objective function. MAC provides a measure of consistency between mode shapes vectors from a finite element analysis and predictions from model updating using Eq. 3.5.

$$min_x \begin{cases} F(x) \le 0 \\ \text{lb} \le x \le \text{ub} \end{cases} \tag{3.1}$$

$$min_x \begin{cases} \frac{1}{2}\,||F(x)||_2^2 = \frac{1}{2}\sum_i f_i\,(x)^2 \\ \text{lb} \le x \le \text{ub} \end{cases} \tag{3.2}$$

$$min_{x,\gamma} \begin{cases} F(x) - w.\gamma \le \text{goal} \\ \text{lb} \le x \le \text{ub} \end{cases} \tag{3.3}$$

$$F(x) = 1\text{-}MAC \tag{3.4}$$

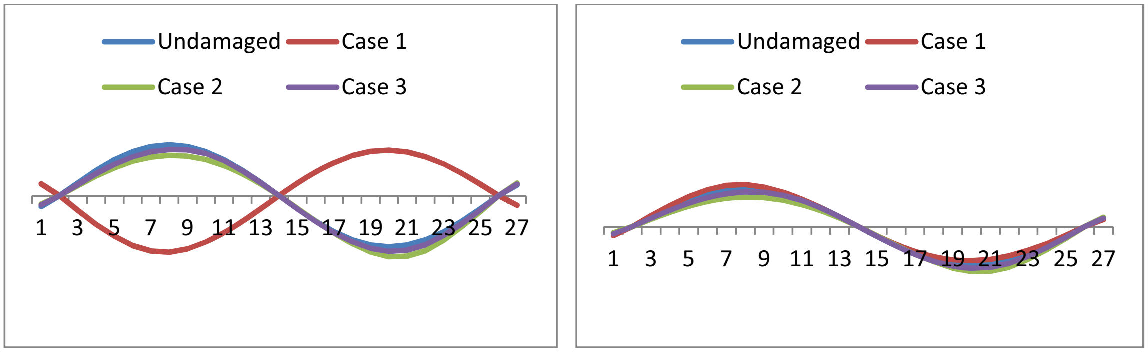
$$MAC(r, q) = \frac{|\{\phi_A\}_r^T\{\phi_X\}_q|}{(\{\phi_A\}_r^T\{\phi_A\}_r)(\{\phi_X\}_q^T\{\phi_X\}_q)} \tag{3.5}$$

$\{\phi_A\}$ is a mode shapes vector of damaged from finite element analysis and $\{\phi_X\}$ is a mode shapes vector of analytical predictions from modal updating process; in Eq. 3.3 w is weighting vector to control the attainment factor of the goals; γ is a slack variable used as a dummy argument in the optimization. In Eq. 3.1, Eq. 3.2 and Eq. 3.3 *lb* and *ub* are the lower and upper bounds of design parameters.

The Modal Scale Factor (MSF) has been applied to mode shape vectors to normalize all estimate vectors that takes into account the magnitude and the phase differences. The mode shapes vectors from a finite element analysis can be compared and contrasted with those from modal updating using Eq. 3.6.

$$MSF\,(cdr) = \frac{\{\Psi_{cr}\}^T\{\Psi_{dr}\}}{\{\Psi_{dr}\}^T\{\Psi_{dr}\}} \tag{3.6}$$

$\{\Psi cr\}$ is modal vector for reference c, mode r and $\{\Psi dr\}$ is modal vector for reference d, mode r. The undamaged mode shapes vector is used as reference vector for MSF.



(a) (b)

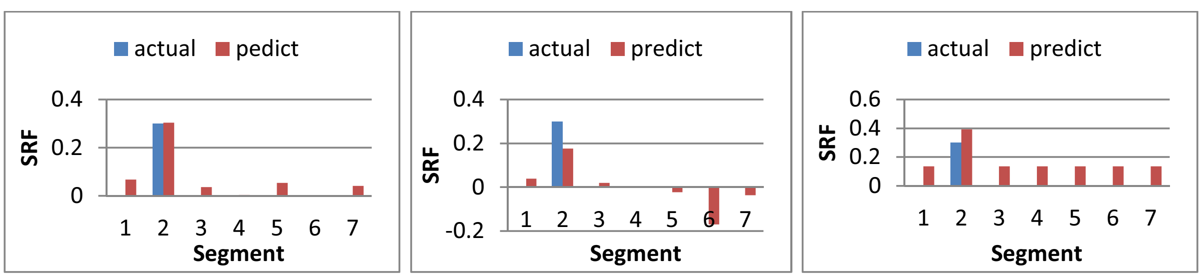
Figure 3: Mode 1 mode shapes in different damage cases (a) before and (b) after MSF

The result from model updating is E value in every segment. The changes in E values are therefore used to detect and locate the damage of the slab structure using stiffness reduction factor (SRF) in Eq. 3.7. To demonstrate the effect on the result improvement by tolerance criteria applied, Mean-Squared Error (MSE) is computed.

$$SRF = \frac{E_{udamaged} - E_{damaged}}{E_{udamaged}} \tag{3.7}$$

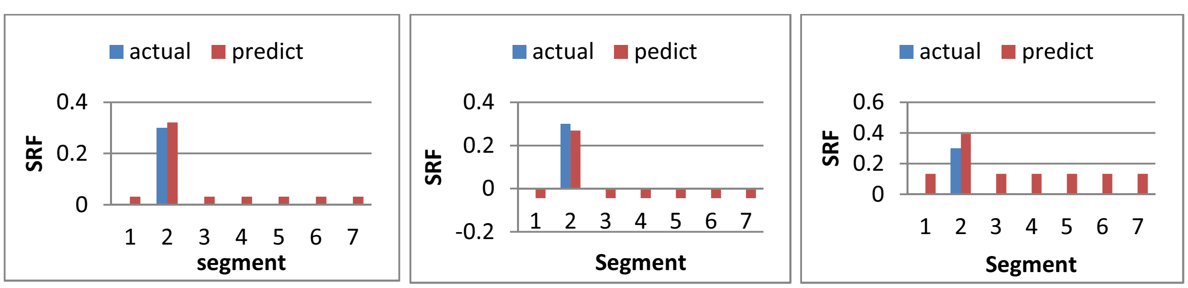
Result and Discussion

To examine the applicability of three different algorithms for damage detection, model updating is performed using *fmincon* function, *lsqnonlin* function and *fgoalattain* function. Figure 4, 6 and 8 show the comparison between actual and predicted SRF value for Case 1, Case 2 and Case 3 respectively. Figure 5, 7 and 9 show the improvement of performance after applying higher tolerance criteria.



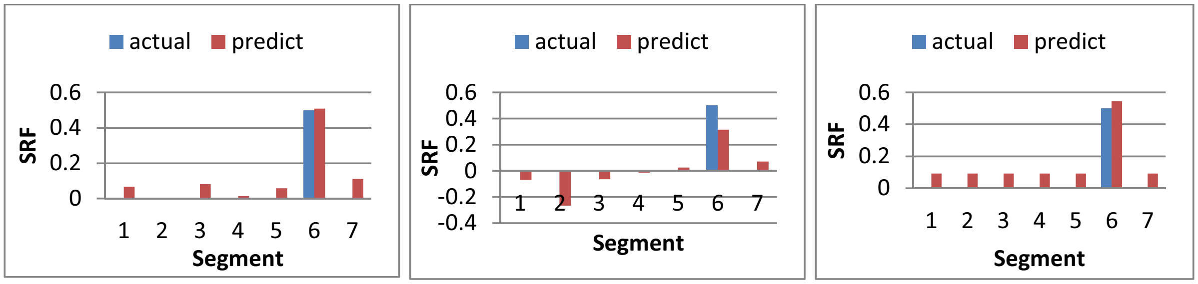
a) *fmincon* function b) *lsqnonlin* function c) *fgoalattain* function

Figure 4: Performance of model updating for Case 1 using different function for default tolerance



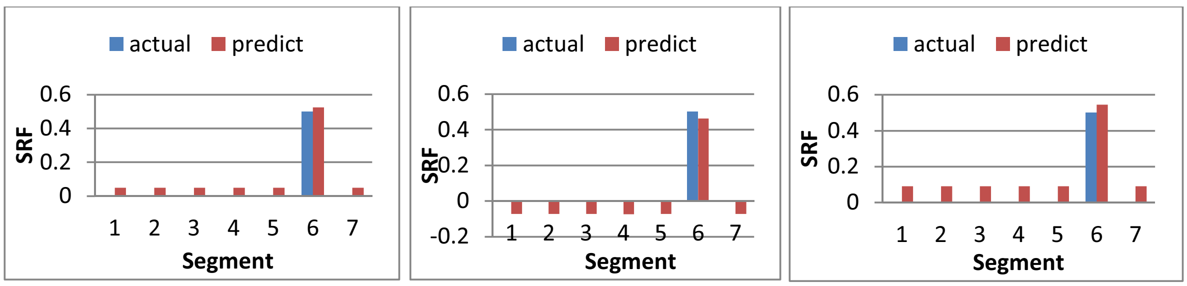
a) *fmincon* function b) *lsqnonli***n** function c) *fgoalattain* function

Figure 5: Performance of model updating for Case 1 using different function for higher tolerance



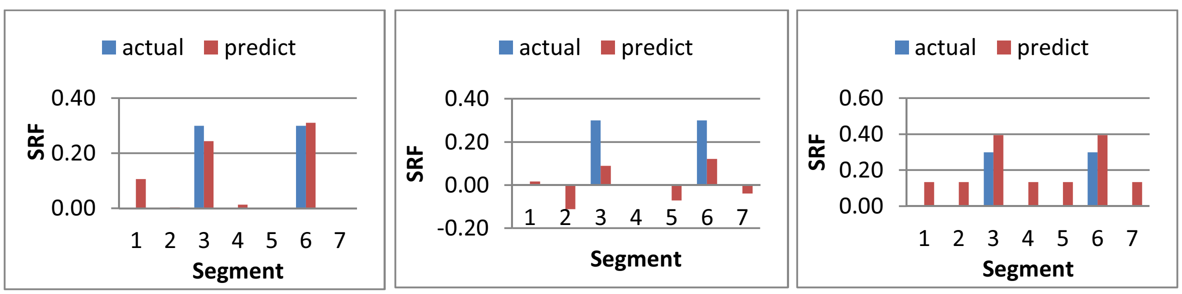
a) *fmincon* function b) *lsqnonlin* function c) *fgoalattain* function

Figure 6: Performance of model updating for Case 2 using different function for default tolerance



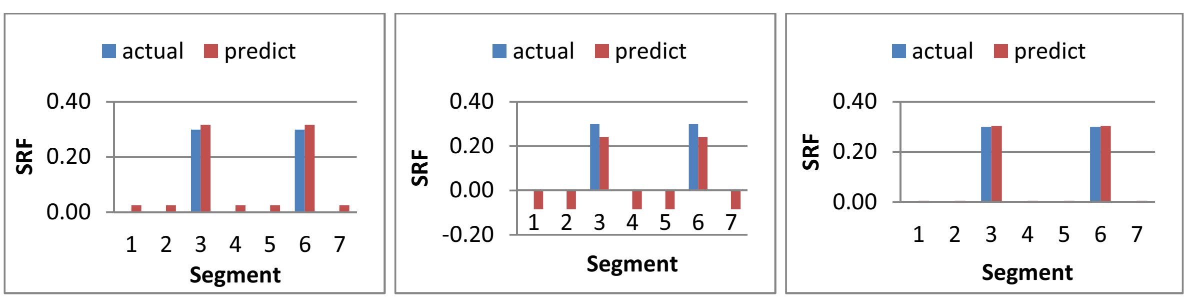
a) *fmincon* function b) *lsqnonlin* function c) *fgoalattain* function

Figure 7: Performance of model updating for Case 2 using different function for higher tolerance



a) *fmincon* function b) *lsqnonlin* function c) *fgoalattain* function

Figure 8: Performance of model updating for Case 3 using different function for default tolerance

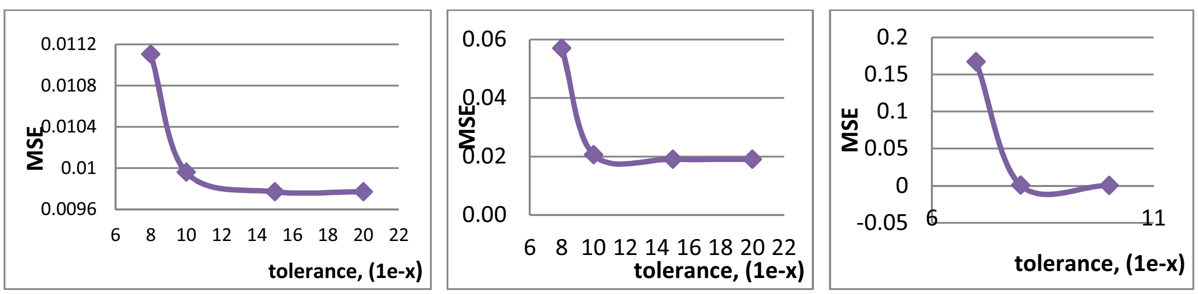


a) *fmincon* function b) *lsqnonlin* function c) *fgoalattain* function

Figure 9:Performance of model updating for Case 3 using different function for higher tolerance

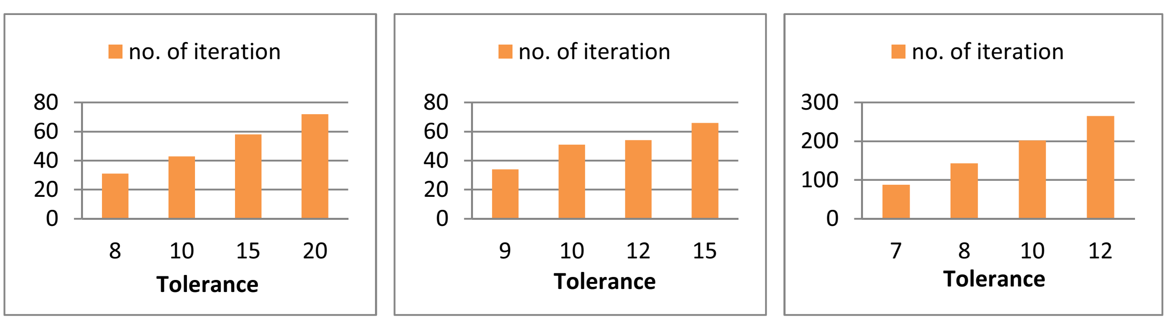
The performance of model updating using different algorithms is evaluated based on the capability to predict the actual damaged comprise the location and severities of the damaged. Based on Figure 4(a), 6(a) and 8(a) it is seen that *fmincon* optimization function can identify the damaged location correctly with small error of damage severities. However, false identification in other undamaged segment occurs in all cases with acceptable error. Using *lsqnonlin* optimization function from Figure 4(b), 6(b) and 8(b) show the location of damaged identified correctly but poorly in damaged severities. The highest error of 21% for Case 3 can be illustrated in Figure 8(b). Yet still can locate the damaged due to the SRF of segment 3 and 6 is higher over other segments which is 0.09 and 0.12 respectively. Figure 4(c), 6(c) and 8(c) demonstrate the ability of *fgoalattain* optimization function to predict damaged location and severities. The locations are successfully identified with slightly overestimate the severities for all damaged cases. However, false identification for all undamaged segments is predicted with constant SRF value. According to [3] underestimates and overestimates of SRF value can be observed at undamaged segments due to numerical error and has been encountered by many researchers before. The trend for *fmincon*, *lsqnonlin* and *fgoalattain* performance is same for single cases (Case 1 and Case 2) and multiple case (Case 3).

Figure 5, 7 and 9 show the improvement of model updating performance after applying higher tolerance for different algorithms in all damaged cases. It can be seen that higher tolerance criteria provide more accurate result in damaged detection as shown in Figure 10 (a)-(c). The performance of different algorithms improve obviously in locating damaged and predict damaged severities for all damage cases as illustrate in Figure 5(a)-(c), 7(a)-(c) and 9(a)-(c). The false identification in undamaged segments with constant SRF value is observed. However, this constant false identification has been noticed earlier using default tolerance in *fgoalattain* function. It indicates *fgoalattain* may provide more accurate result without applying higher tolerance. This is parallel to the study by [4] which using *fgoalattain* function rather than others function for model updating process due to ability to give better result. Besides, the effect of higher tolerance applied is the increment of number of iteration as shown in Figure 11(a)-(c). The same trend is discovered for all three different functions in which increment of number of iteration with higher tolerance applied.



a) MSE for *fmincon* function b) MSE for *lsqnonlin* function c) MSE for *fgoalattain* function

Figure 10: MSE for different function for multiple damaged cases



(a) *fmincon* (b) *lsqnonlin* (c) *fgoalattain*

Figure 11: Tolerance vs Number of iteration

Conclusion

This study demonstrates the comparative study of different algorithms using different optimization function which are constrained optimization (*fmincon*), least-square optimization (*lsqnonlin*) and multiobjective optimization (*fgoalattain*) in damage detection. Three damages cases were investigate to verify the applicability of model updating to detect damaged in slab structure. Single damaged and multiple damaged is simulated using FEM and dynamic parameter of mode shapes obtained from FE analysis is employed in this study. Higher tolerance is applied to all optimization function to demonstrate the effect of tolerance on the result performance. The conclusions that can be made based on the result are as follow:

1. Model updating using three optimization functions of different algorithms used in this study able to provide reliable result in damaged detection. However, the performance of *fgoalattain* optimization function using default tolerance provides better result compare to *lsqnonlin* optimization function and *fmincon* optimization function.
2. Dynamic properties of mode shapes generated from FEM utilize in evaluation of MAC in a function of *E* value may provide a good result in damaged detection. However, study by [3] and [4] show that the accuracy of the result can be improved by combination of natural frequencies and mode shapes in evaluation of objective function.
3. It is observed that higher tolerance criteria can improve the performance of all the different optimization functions in damage detection to detect damage location and predict the severities of the damaged which can illustrate in MSE graph versus tolerance for single and multiple damaged cases. The decrement of the MSE value indicates the result is improved as the tolerance is increased.

References

[1] Charles R. Farrar,Scott W. Doebling and David A. Nix. 2001. “Vibration-based structural damage identification.” Journal of Mathematical, Physical & Engineering Sciences, 131-149

[2] J. E. Mottershead, M. Link, M. I. Friswell, “The sensitivity method in finite element model updating: A tutorial, Mechanical Systems and Signal Processing”, Vol. 25, Issue 7, pp 2275–2296, 2011.

[3] Norhisham Bakhary, Structural Condition Monitoring and Damage Identification with Artificial Neural Network, University of Western Australia, PhD. Thesis, 2008.

[4] Saerehati Umar, “Response Surface Methodology for Damage Detection Using Frequency and Mode Shapes”, Universiti Teknologi Malaysia, Master Thesis, 2015.

[5] S. N. Zahari, “A Review on Model Updating of Joint Structure for Dynamic Analysis Purpose, Unuversiti Tun Hussein Onn”, Master Thesis, (n.d).

[6] Scott W. Doebling, Charles R. Farrar and Michael B. Prime, “A Summary Review of Vibration-Based Damage Identification Methods”,(n.d).

[7] E. Peter Carden and Paul Fanning, “Vibration Based Condition Monitoring: A Review”, Journal of Structural Health Monitoring, 3(4),355-357,2004.

[8] Rytter A., “Vibration based inspection of Civil engineering structure”, PhD. Dissertation, Department of Building Technology and Structural Engineering, Aalborg University, Denmark, 1993.

[9] C. P. Fritzen, D. Jennewein,and T. Kieffer, “Damage Detection Based on Model Updating Methods”, Journal of Mechanical System and Signal Processing, 12(1), 163:186, 1998.

[10] J. E. Mottershead, M. Link, M. I. Friswell, “Model Updating in Structural Dynamics: A Survey”, Journal of Sound and Vibration, 167(3), 347:375, 1993.

[11] Bernd Caesar and Jorg Peter, “Direct Update of Dynamic Mathematical Models from Modal Test Data”, AIAA Journal, 25(11), 1987.

[12] J. E. Mottershead, M. Link, M. I. Friswell, “The Sensitivity Method in Finite Elemet Model Updating: A tutorial”, Journal of Mechanical System and Signal Processing, 2275:2296, 2011.

[13] Jaishi, B., and Ren, W. X. “Finite Element Model Updating Based on Eigenvalue and Strain Energy Residuals Using Multiobjective Optimisation Technique” Mechanical Systems and Signal Processing 21(5): 2295-2317, 2007.
[14] Y. X. Zhang, S.H. Sim and B.F. Spencer, “Finite Element Model Updating Of A Truss Using Incomplete Modal Data”, Shanghai Normal University and University of Illinois, Urbana, USA.
[15] Zhang, Q. W., Chang, C. C., and Chang, T. Y. P., “Finite Element Model Updating for Structures with Parametric Constraints”, Earthquake Engineering & Structural Dynamics 29(7): 927-944, 2000.
[16] Xianfei He, “Vibration-based Damage Identification and Health Monitoring of Civil Structures”, e-book, 2008.
[17] Farid Asma and Amar Bouazzouni, “Least Squares Finite Element Model Updating”, University Mouloud Mammeri, 2005.
[18] Pelin G. B., Edwin Reynders and Guido De Roeck, “An improved finite element model updating method by the global optimization technique ‘Couple Local Minimizer’”, Istanbul Technical University, 2007.