EXPLORATION OF METHODS FOR SLOPE STABILIZATION
INFLUENCE BY UNSATURATED SOIL

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Abstract: This study will lead to the analysis of unsaturated soil using Bishop’s Simplified method which is one method to analyse slope stability in method of slices. In this study, the original formula for Bishop’s Simplified method of saturated soil were modified by adding the element of matric suction, together with unsaturated friction angle, which was applicable for the analysis of unsaturated soil. In this study, 0 kPa and 20 kPa of matric suctions were applied in the analysis for both methods. From the analysis, the result indicates that the factor of safety (FOS) value of Bishop’s Simplified method was 8.82 % higher than Fellenius’s method for 0 kPa suction, which means that the soil is in saturated condition. For 20 kPa of suction, the FOS of Bishop (1955) was 6.84 % higher than Fellenius (1936). It can be concluded that, the reason for the relative accuracy of the Bishop’s Simplified method is that in considering only the vertical equilibrium of any slice, there is no need to account for the horizontal components of the inter-slice forces.

Keywords: Soil suction, Bishop, unsaturated soils, Fellenius, factor of safety

1.0 Introduction

Nowadays, slope failure can be consider as one of the mostfrequent disaster that happened not only in Malaysia, but also in other countries. This is due to the increment and rising of development all over the world wether for developed or other countries which may lead to extensively cutting the existence slope during the development. According to Sutejo and Gofar, (2015) failure occurs of man-made slope is caused by designs errors including geometric design i.e. slope inclination, slope height, and the inability to determine the load that may affect the slope together with the soil resistance. This study aims to determine the factor of safety (FOS) of unsaturated soil slopes by using one method from method of slices which is Bishop’s Simplified method (Bishop, 1955). The original formula of Bishop’s Simplified method for saturated soil was modified in order to include the element of matric suction, \( \mu_a - \mu_w \) together with
unsaturated friction angle, \( \phi^b \). The FOS that been determined from the calculation using Bishop’s Simplified method then was analysed and finally, a comparison of FOS between Bishop (1955) with Fellenius (1936) was carried out in order to determine which method gave higher and more accurate FOS for slope stabilization.

2.0 Methodology

In the current work, a reasonably simple framework has been sought that will permit the first assessment of the influence of soil suction changes on soil shear strength. For this purpose, the following relationship provided by (Fredlund et al., 1978) appears suitable:

\[
\tau = c' + (\sigma_n - \mu_u) \tan \phi' + (\mu_u - \mu_w) \tan \phi^b
\]

(1)

where \((\mu_u - \mu_w)\) is the matric suction and \( \phi^b \) is the angle indicating the rate of increase in shear strength relative to matric suction. \((\sigma_n - \mu_u)\) is the net normal stress, \(c'\) is the effective cohesion and \( \phi' \) is angle of friction.

Fredlund and Rahardjo (1993) provided the relationship on how shear strength, matric suction together with net normal stress give a three dimensional failure surface, as shown in Figure 1.

![Figure 1: Extended Mohr-Coulomb failure envelope for unsaturated soils, modified after Fredlund and Rahardjo (1993)](image-url)
Figure 2 show the forces acting on a slice within the sliding soil mass.

![Figure 2: Forces acting on a slice through a sliding mass with a circular slip surface, modified after Fredlund and Rahardjo (1993)](image)

To calculate the FOS in unsaturated soil slope, a force equation which includes matric suction must be established. The mobilized shear force, $S_m$, at the base of a slice can then be written as (Lambe and Whitman, 1969).

$$S_m = \frac{\tau l}{F}$$

(2)

Where $\tau$ is shear strength of unsaturated soil as defined previously in Equation (1). Combining Equations (1) and (2), gives,

$$S_m = \frac{l \left( c' + \frac{\sigma_a - \mu_a}{\mu_a - \mu_a} \tan \phi' + \frac{\mu_a - \mu_a}{\tan \phi'} \right)}{F}$$

(3)

Resolve Bishop vertically,

$$N \cos \alpha = W + \Delta X - S \sin \alpha$$

$$N = \frac{W + \Delta X - S \sin \alpha}{\cos \alpha}$$
\[
S = \frac{(c' l + (N - \mu_a l) \tan \phi' + (\mu_a - \mu_w) l \tan \phi^b)}{F}
\]  
(4)

Substitute for \( N \):
\[
S = \frac{(c' l \cos \alpha + (W + \Delta X - S \sin \alpha - \mu_a l \cos \alpha) \tan \phi' + (\mu_a - \mu_w) l \cos \alpha \tan \phi^b)}{F \cos \alpha}
\]  
(5)

As \( b = \text{width of slice} = l \cos \alpha \) and substitute \( (\mu_a - \mu_w) \) which is matric suction as \( M \) and also assuming the air pore pressure is constant (atmospheric) then \( \mu_a = 0; \)
\[
S = \frac{1}{F} \left[ \left( c' b + (W + \Delta X - \mu_a b) \tan \phi' + M b \tan \phi^b \right) \cos \alpha \right] - S \tan \alpha \tan \phi'
\]  
(6)

Substitute \( 1 + \frac{\tan \alpha \tan \phi'}{F} = m_a; \)
\[
S = \frac{1}{F} \left( c' b + (W + \Delta X - \mu_a b) \tan \phi' + M b \tan \phi^b \right) \frac{1}{m_a}
\]  
(7)

Moment of equilibrium;
\[
\sum W \sin \alpha = \sum S
\]

\[
F = \frac{\sum \left[ (c' b + (W + \Delta X - \mu_a b) \tan \phi' + M b \tan \phi^b) \left( \frac{\sec \alpha}{m_a} \right) \right]}{\sum W \sin \alpha}
\]  
(8)

After a lot of consideration, the final formula is as stated in Equation (8). The element of matric suction, \( (\mu_a - \mu_w) \) together with unsaturated friction angle, \( \phi^b \) was included in the original equation of Bishop’s Simplified method of saturated soil. When suction becomes zero, it means that the soil is saturated and the equation will turn to the original equation as done by Bishop.
3.0 Results and Discussion

Figure 3 shows method of slices: division of sliding mass into slices and forces acting on a typical slice.

Chowdhury et al., (2010) has reported that, the major difference between Bishop’s Simplified method with Fellenius’s method (Fellenius, 1936) is that in considering the vertical equilibrium of any slices, there is no need to account for the horizontal components of the inter-slice forces. The resolution of forces takes place in vertical direction instead direction normal to the arc. Meaning that, with Bishop’s Simplified method of slices, the side forces $E$ acting on the sides of the slices will not enter into the analysis. It is assumed that the shear side forces $X$ may be neglected without introducing serious error into the analysis.

Figure 4 shows the detail of slope geometry with slip surface and location of slices by Ishak (2014). Ishak (2014) used this detail geometry in his research to calculate slope stabilization using Fellenius’s method (1936) equation for unsaturated soil which had been modified by Rees and Ali (2012).
The experimental values of shear strength with $\phi^b$ angle of tropical residual soil suggested by Ishak (2014) is as shown in Table 1. In this study, two suction values were used (0 kPa and 20 kPa) for FOS values on slope. Table 2 shows the calculations of Bishop’s Simplified method with 0 kPa suction.

Table 1: Experimental values of shear strength with $\phi^b$ angle of tropical residual soil

<table>
<thead>
<tr>
<th>Researcher</th>
<th>Location</th>
<th>$c'$ (kPa)</th>
<th>$\phi$ (°)</th>
<th>$\phi^b$ (°)</th>
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<tr>
<td>Ishak (2014)</td>
<td>Faculty of Electrical Engineering, UTM</td>
<td>9</td>
<td>23</td>
<td>20</td>
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</table>

Ishak (2014) suggested that the type of soil in Faculty of Electrical Engineering, UTM is sandy silt with cohesion value, $c$ is 9 kPa, friction angle, $\phi$ is 23°, and unsaturated friction angle, $\phi^b$ is 20°.
Table 2: Calculations of Bishop’s Simplified method with 0 kPa suction
(values for z, b, W, and α are suggested by Ishak, 2014)

<table>
<thead>
<tr>
<th>Slice No</th>
<th>z (cm)</th>
<th>b (m)</th>
<th>W (kN)</th>
<th>α (°)</th>
<th>sin α</th>
<th>tan α'</th>
<th>tan β</th>
<th>tan α</th>
<th>cos α</th>
<th>c b tan α'</th>
<th>W tan α'</th>
<th>assumed F OS</th>
<th>assumed F OS</th>
<th>assumed F OS</th>
<th>assumed F OS</th>
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<th>(4-1)</th>
<th>fs = 1.85</th>
<th>(4-2)</th>
<th>fs = 1.9</th>
<th>(4-3)</th>
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<th>(kN)</th>
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<td>0.000</td>
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<td>0.424</td>
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<tr>
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</table>

FOS₁ = \frac{270.085}{146.170} = 1.85

FOS₂ = \frac{270.854}{146.170} = 1.85

FOS₃ = \frac{271.590}{146.170} = 1.86

FOS₄ = \frac{272.297}{146.170} = 1.86

Figure 5 shows the graph of Bishop’s Simplified method (1955) with 0 kPa suction.
Table 3: Differences of FOS value with 0 kPa suction

<table>
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<tr>
<th>Type of Analysis</th>
<th>FOS</th>
<th>Percentage Difference (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fellenius’s method (1936) (Ishak, 2014)</td>
<td>1.70</td>
<td>0</td>
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<tr>
<td>Bishop’s Simplified method (1955)</td>
<td>1.85</td>
<td>8.82</td>
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The results suggested that, calculation by using Bishop’s Simplified method gave higher FOS value compare to ordinary Fellenius’s method (1936) by 8.82 % for 0 kPa suction. This is due to different consideration from both methods which Fellenius’s method only consider moment of equilibrium while Bishop’s method consider both moment of equilibrium and vertical forces. This make Bishop’s method more accurate compare to Fellenius’s method thus give higher FOS value. Table 4 shows the calculations of Bishop’s Simplified method with 20 kPa suction.
Table 4: Calculations of Bishop’s Simplified method with 20 kPa suction
(values for $z, b, W,$ and $\alpha$ are suggested by Ishak, 2014)

<table>
<thead>
<tr>
<th>Slice No</th>
<th>$z$ (cm)</th>
<th>$b$ (m)</th>
<th>$W$ (kN)</th>
<th>$\alpha$ ($^\circ$)</th>
<th>$\sin \alpha$</th>
<th>$\tan \phi'$</th>
<th>$\tan \phi_b$</th>
<th>$\tan \alpha$</th>
<th>$\cos \alpha$</th>
<th>$c_b$</th>
<th>$W \tan \phi'$</th>
<th>$y_b \tan \phi_b$</th>
<th>$\phi'$</th>
<th>$\gamma$ (kN/m$^3$)</th>
<th>$\psi$ (kPa)</th>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>364.094</td>
<td>364.707</td>
<td>365.300</td>
<td>365.673</td>
<td>146.170</td>
</tr>
</tbody>
</table>

$\text{FOS}_1 = \frac{364.094}{146.170} = 2.49$

$\text{FOS}_2 = \frac{364.707}{146.170} = 2.50$

$\text{FOS}_3 = \frac{365.300}{146.170} = 2.50$

$\text{FOS}_2 = \frac{366.873}{146.170} = 2.50$

Figure 6 shows the graph of Bishop’s Simplified method with 20 kPa suction.
The graph indicates that, the actual FOS value for Bishop (1955) with 20 kPa suction is 2.50. Since the FOS is greater than 1, therefore it is safe. Table 5 shows the percentage differences of FOS between Bishop’s Simplified method with Fellenius’s method of 20 kPa suction.

<table>
<thead>
<tr>
<th>Type of Analysis</th>
<th>FOS</th>
<th>Percentage Difference (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fellenius’s method (1936)</td>
<td>2.34</td>
<td>0</td>
</tr>
<tr>
<td>(Ishak, 2014)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bishop’s Simplified method (1955)</td>
<td>2.50</td>
<td>6.84</td>
</tr>
</tbody>
</table>

The results indicate that, calculation by using Bishop’s Simplified method gave higher FOS value compare to ordinary Fellenius’s method (1936) by 6.84 % for 20 kPa suction. Clearly, these two comparisons show that more accurate FOS value for slope stabilization can be obtained by calculating using Bishop’s Simplified method compare to Fellenius (1936). Also, as the FOS value was greater than 1, therefore, the slope was in safe and good condition.
4.0 Conclusion

It can be concluded that, Bishop’s Simplified method gave higher and more accurate FOS value compare to Fellenius’s method (1936) for slope stabilization. The results indicate that, there is more than 8% differences in FOS when calculated using Bishop (1955) instead of Fellenius (1936) for 0 kPa suction. Bishop (1955) also gave more than 6% differences of FOS value compare to Fellenius (1936) when applying 20 kPa suction value. The analysis of Bishop’s simplified method was carried out in term of stresses instead of forces which were used in Fellenius (1936). The major difference between these two methods is that, in Bishop (1955), the resolution of forces takes place in the vertical direction instead the direction normal to the arc. Meaning that, the side forces $E$ of Bishop acting on the sides of the slices will not enter into the analysis. The reason for the relative accuracy of the Bishop (1955) is that in considering only the vertical equilibrium of any slice, there is no need to account for the horizontal components of the inter-slice forces (Chowdhury et al., 2010).

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References


