

ANALYSIS AND DESIGN OF HAMMERHEAD BRIDGE PIER  
USING STRUT AND TIE METHOD.

ABDUL KADIR BIN AHYAT

UNIVERSITI TEKNOLOGI MALAYSIA

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JALAN NAKHODA KANAN  
KG. NAKHODA, 68100 BATU CAVES  
SELANGOR D.E.ASSC. PROF. M. DR. WAHID OMARTarikh: 11-4-05Tarikh: 4-4-05

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“ I hereby declare that I have read this project report and in my opinion this project report is sufficient in terms of scope and quality for the award of the degree of Master of Engineering (Civil – Structure)

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:



Name of Supervisor

:

ASSC. PROF. IR. DR. WAHID BIN OMAR

Tarikh

:

4-4-05


ANALYSIS AND DESIGN OF HAMMERHEAD BRIDGE PIER  
USING A STRUT AND TIE METHOD.

ABDUL KADIR BIN AHYAT

A project report submitted in partial fulfillment of the  
requirements for the award of the degree of  
Master of Engineering (Civil – Structure)

Faculty of Civil Engineering  
Universiti Teknologi Malaysia

I declare that this entitled “ANALYSIS AND DESIGN OF HAMMERHEADBRIDGE PIER USING STRUT AND TIE METHOD” is the result of my own research except as cited in the references. The project report has not been accepted for any degree and is not concurrently submitted in candidature of any other degree.

Signature :   
Name : ABDUL KADIR BIN AHYAT  
Date : 04 April 2005.

DEDICATION

TO MY BELOVED PARENT,  
HAJI AHYAT BIN MD. NOR  
AND  
HAJAH KAMSIAH BTE BERNEH

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## ABSTRACT.

The main advantages of truss model are their transparency and adaptability to arbitrary geometric and loading configuration. In strut-and-tie modeling, the internal stresses are transferred through a truss mechanism. The tensile ties and compressive struts serve as truss members connected by nodal zones. The advantages have been thrust into the back ground by several recent developments of design equations based on truss models,

The present study is focus on developing a uniform design procedure for applying the strut-and-tie modeling method to hammerhead pier. A study was conducted using hammerhead piers that were previously designed using the strength method specified by code. This structure was completed and had put into service. During the inspection, cracks were observed on the piers. The scope of this study is to highlight the application of a newer generation strut-and-tie model, which is not practice at the time of the original design. Depth to span ratios varies from 1.5 to 2.11 and the girders are transferring loads very close to the support edge, making these hammerheads ideals candidates for strut-and-tie application. This study only focus on comparison the reinforcement detail drawing produce previously designed using the strength method, and reinforcing requirement using strut-and-tie model.

Based on the design studies, a well-defined procedure for designing a hammerhead pier utilizing the strut-and-tie model was established that may be used by bridge engineers.

There could be numerous reasons for the crack to develop. Shrinkage, stress concentration or some erection condition may be a few of them.



## **ABSTRAK.**

Kelebihan model “strut and tie ” ia ketelusan melihat kerangka yang di cadangkan dan memudahkan melihat dan meramalkan kedudukan beban yang dikenakan terhadap struktur yang di cadangkan.

Analisis mengikut model “strut and tie ” menggunakan kaedah kekuatan mampatan dan kaedah kekuatan tegangan yang saling bertindak diantara satu sama lain hasil daripada ikatan disetiap nod. Kebaikan analisis menggunakan kaedah kekuatan mampatan dan kekuatan tegangan yang saling bertindak diantara mereka telah membuat pengkaji cuba membangunkan kaedah rekabentuk berpandukan kaedah model “strut and tie model”.

Kajian ini menjurus untuk memajukan satu kaedah yang setara untuk merekabentuk menggunakan kaedah model “strut and tie ” untuk tiang Jambatan berbentuk T. Kajian ini dikendalikan menggunakan struktur tiang jambatan berbentuk T yang telah direkabentuk terlebih dahulu menggunakan analisa kekuatan lentur mengikut keperluan amalan rekabentuk.

Struktur ini telah siap dibina dan dibuka untuk kegunaan lalulintas. Semasa pemerhatian terhadap struktur tersebut didapati ada beberapa rekahan di permukaan dinding struktur. Bidang kajian ini adalah untuk menunjukkan penggunaan analisis model “strut and tie model” yang masih dalam peringkat pembangunan boleh diguna pakai untuk mereka bentuk struktur tersebut. Nisbah ketinggian dinding tembok dan panjang rasuk adalah berbeza diantara 1.5 hingga 2.11 dan beban yang terletak diatas rasuk tersebut, hampir dengan kedudukan tiang rasuk, ini membuatkan struktur tersebut amat sesuai untuk dianalisis menggunakan kaedah analisis model “strut and tie ”.

Hasil daripada kajian rekabentuk ini, satu kaedah rekabentuk menggunakan tindak balas struktur “strut and tie ” dapat dimajukan untuk dicadangkan untuk merekabentuk struktur tiang jambatan berbentuk T, yang mana boleh digunakan oleh Jurutera Jambatan.

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## LIST OF SYMBOLS

$a$	=	depth of the compression block
$A_s$	=	the required area of steel
$A_c$	=	cross sectional area at the end of Strut
$A_n$	=	area of a Nodal Zone face in which the force is framing, measured perpendicular to the direction of the force.
$b$	=	width of concrete section
$b_w$	=	the width of web
$d$	=	depth from extreme compression fibres to reinforcing steel
$D$	=	depth of the nodal zone
$D_A$	=	available effective depth
$D_R$	=	Required effective depth
$f'_c$	=	concrete compressive strength.
$f_{cu}$	=	effective compressive strength and
$f_y$	=	the tie yield strength
$F_i$	=	force in strut or tie $i$
$F_n$	=	nominal strength of Strut, Tie, or Node, and
$F_u$	=	factored force demand of the Strut, Tie, or Node.
$l_i$	=	length of member $i$
$M_n$	=	nominal moment capacity
$N_u$	=	the factored tie force
$P_n$	=	nominal resistance of strut or tie
$P_u$	=	ultimate capacity of strut or tie
$V_c$	=	the nominal shear strength provided by the concrete
$V_n$	=	the factored shear force at the section considered
$W$	=	width of the nodal zone

$\beta_s$	=	1.00 for prismatic Struts in uncracked compression zones,
$\beta_s$	=	0.04 for Struts in tension members,
$\beta_s$	=	0.75 if Struts may be bottle shaped and crack control reinforcement is included,
$\beta_s$	=	0.60 if Struts may be bottle shaped and crack control reinforcement is not included, and
$\beta_s$	=	0.60 for all other cases.
$\beta_n$	=	1.00 if Nodes are bounded by Struts and/or bearing areas,
$\beta_n$	=	0.80 if Nodes anchor only one Tie, and
$\beta_n$	=	0.60 if Nodes anchor more than one Tie.
$\phi$	=	strength reduction factor,
$\varepsilon_{mi}$	=	mean strain of member $i$
$\rho_{vi}$	=	steel ratio of the $i$ -th layer of reinforcement crossing that strut
$\gamma_i$	=	angle between the axis of a strut and the bars



## **CHAPTER 1**

### **INTRODUCTION**

#### **1.1 Introduction**

Strut-and-tie modeling is an analysis and design tool for reinforced concrete elements in which it may be assumed that internal stresses are transferred through a truss mechanism. The tensile ties and compressive struts serve as truss members connected by nodal zones. The internal truss, idealized by the strut-and-tie model, implicitly account for the distribution of both flexure and shear.

#### **1.2 Problem Statement**

Three procedure are currently used for the design of load transferred members such as deep beams:

- ❖ Empirical design method
- ❖ Two or three dimensional analysis, either linear or nonlinear
- ❖ By mean of trusses composed of concrete struts and steel tension ties.

Strut and tie model is considered a rational and consistent basis for designing cracked reinforced concrete structure. It is mainly applied to the zones where the

beam theory does not apply, such as geometrical discontinuities, loading points, deep beams and corbels.

The main advantage of truss model are their transparency and adaptability to arbitrary geometric and loading configuration. In strut-and-tie modelling, the internal stresses are transferred through a truss mechanism. The tensile ties and compressive struts serve as truss members connected by nodal zones. The advantages have been thrust into the background by several recent developments of design equations based on truss models,

In 1998, the AASHTO LRFD Bridge Specifications (1998) incorporated the strut and tie modeling procedure for the analysis and design of deep reinforced concrete members where sectional design approaches are not valid. In most instances, hammerhead piers can be defined as deep reinforced concrete members and therefore, should be designed using the strut-and-tie modeling approach. However, most bridge engineers do not have a broad knowledge on the strut-and-tie model due to the unfamiliarity with the design procedure. Therefore, it is likely that, with the formulation of a well-defined strut-and-tie modeling procedure, practicing engineers will become more comfortable with the design method and therefore, employ the method more often and consistently.

The successful application of a strut-and-tie model depends on a reliable visualization of the path of the force flows. In a typical strut-and-tie analysis, the force distribution is visualised as compressive struts and tensile ties, respectively.

### **1.3 Objectives**

The specific objectives of the study are:

- ❖ To ascertain the degree of strut-and-tie modeling implementation.
- ❖ To compare the flexure and shear reinforcing requirements for typical hammerhead type bridge piers using both strut-and-tie modeling and standard sectional design practices, and
- ❖ To develop a uniform design procedure for employing strut-and-tie modeling for hammerhead piers.

Most codes of practice use sectional methods for designed of conventional beams under bending and shear. ACI building Code 318M-95 assumes that flexure and shear can be handle separately for the worst combination of flexure and shear at a given section. The interaction between flexure and shear is addressed indirectly by detailing rules for flexural reinforcement cutoff point.

### **1.4 Scope of Study**

In these study pier caps was designed using the strut-and-tie modeling procedure and the results compared to the results of the sectional design method. By comparing the results, the reduction or increase in the flexural steel and the shear steel can be quantified.

These new procedure can provide rational and safe design framework for structural concrete under combined actions, including the effects of axial load, bending and torsion.

In addition specific checks on the level of concrete stresses in the member are introduced to ensure sufficient ductile behavior and control of diagonal crack widths at service load level.



## **CHAPTER 2**

### **LITERATURE REVIEW**

#### **2.1 INTRODUCTION**

The strut and tie models have been widely used as effective tools for designing reinforced concrete structures. The idea of a Strut-and-Tie Model came from the truss analogy method introduced independently by Ritter [1] and Morsch [2] in the early 1900s for shear design. This method employs so called Truss Models as its design basis. The model was used to idealised the flow of forced in a cracked concrete beam. In parallel with the increasing availability of the experimental results and the developement of limit analysis in the plastcity theory, the truss analogy method has been validated and improved considerably in the form of full member or sectional design procedures. The Truss Model has also been used as the design basis for torsion.

Later, Schlaich, et al [3] worked to combined individual research conducted on various reinforced concrete elements in such a fashion that Strut-and-Tie modeling could be used for entire structure.

Strut-and-Tie modeling is an analysis and design tool for reinforced concrete elements in which it may be assumed that flexural and shearing stresses are tranferred internally in a truss type member comprised of concrete compressive struts and steel reinforcing tension ties. It should be noted that while the shear design is theoritically couple with the truss model, in most instances

designers perform a separate check for providing additional stirrup type shear reinforcement.

Several theoretical and experimental studies had been carried out to analyse the phenomenon of the shear failure of reinforced concrete beams. During the past few years design codes ACI [4] and AASHTO [5] have adopted Strut-and-tie principles for the design deep beam members. The definition of deep section provided by these specification classifies most hammerhead piers as deep beam.

This literature review is conducted to establish the state of knowledge with regard the possible crack to the hammerhead bridge. The argument has been arise on theoretical method which are most applicable to this type of structure. Strut-and-tie modeling is an analysis and design tool for reinforced concrete which are most suitable for the hammerhead bridge pier but a comparison must be made with beam theory in order to make a comparison with the actual behaviour of the structure . A comparison will be made on the analytical model on the design the hammerhead piers using the strength design method as specified by the standard specification in order to evaluate strut-and-tie modeling. This study will help to focus on developing design procedure for applying to hammerhead bridge pier.

## **2.2 Overview of Strut-and-Tie Modeling**

Strut-and-Tie Method (STM) has been used for several years in Europe and had been included in the AASHTO LRFD [5] Bridge Specification since 1994, it is a new concept for many structural engineers, recommendation for the used of STM to design reinforced concrete members were discuss by previous researchers. In selecting the appropriate design approach, focused on understanding the internal distribution of forces in a reinforced concrete structure and have defined two specific regions; B-Regions and D-Regions as shown in Figure 2.1. The B-Regions of a structure (where B stands for Beam, Bending, or

Bernoulli Beam theory may be employed) have internal states of stress that are easily derived from the sectional forces e.g. bending, shear, etc.

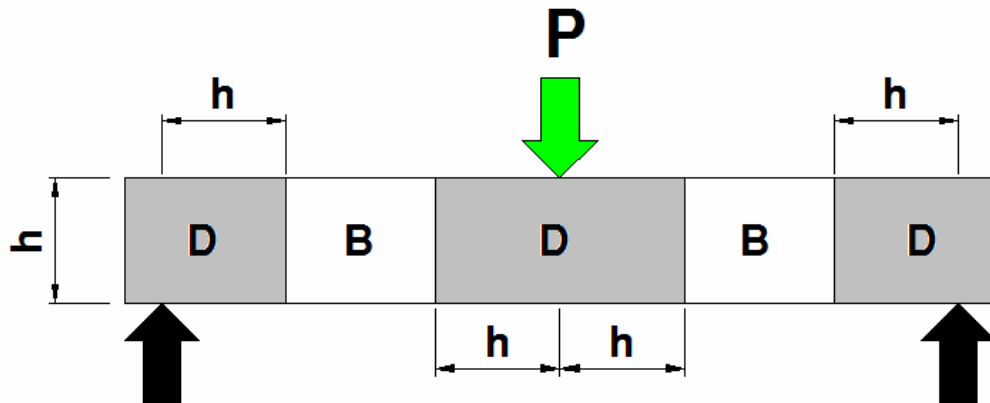


Figure 2.1 ( B-Region and D-Region)

For structural members that do not exhibit plane strain distribution, e.g. the strain distribution is non-linear, the sectional force approach is not applicable. These regions are called D-Regions (where D stands for discontinuity, disturbance, or detail). The D-Regions of a structure are normally corners, corbels, deep sections, and areas near concentrated loads. When D-Regions crack the treatments used such as "detailing," "past experience," and "good practice" often prove inadequate and inconsistent Schlaich, et al [3].

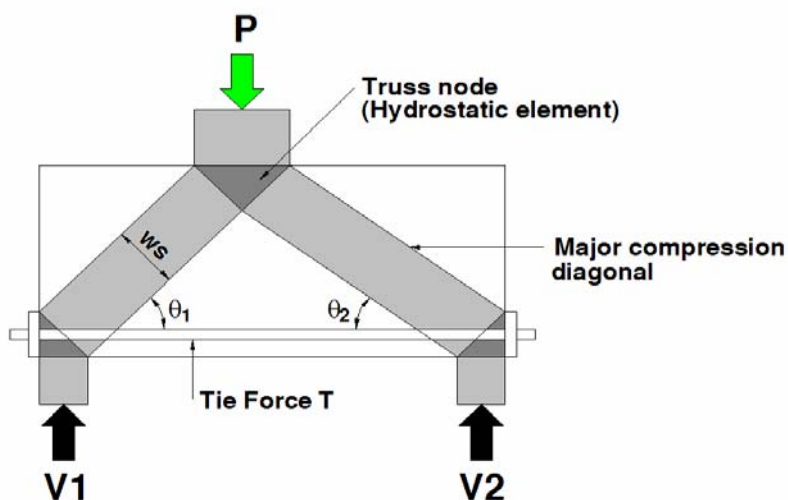


Figure 2.2

ACI [4] Section 10.7.1 For Deep Beam: ACI Section 11.8

For  $L/d < 5/2$  for continuous span For  $L/d < 5$  Shear requirement

For  $L/d < 5/4$  for simple span

Figure 2.2 provided a simple strut-and-tie model applied to a simply supported deep beam. In this figure, the lighter shaded region represent concrete compressive struts, the steel reinforcing bar represent a tensile tie, and the dark shaded regions represent nodal zones.

The tension ties in the truss model may represent one or several layers of flexural reinforcement in the deep section. The locations of the tension ties normally are defined at the centroid of reinforcing mat.

### **2.3 Adequate Selection of Truss Members**

The successful application of a strut-and-tie model depends on a reliable visualization of the paths of force flow. In a typical strut-and-tie analysis, the force distribution is visualized as compressive and tensile force flows that are modeled as compressive struts and tensile ties.

The engineering judgment and an iterative procedure required to produce an adequate reinforcement pattern for a given member. The process of defining the truss begins by defining the flow of forces in the member and locating the nodal zones at points where the external loads act and the loads are transferred between structural members, e.g. the pier cap to pier column or at the supports. The tension ties and compression struts can then be located once the nodal zones have been defined.

The tension ties are located at the assumed centroid of tensile reinforcing beginning and terminating at nodal zones. The compression struts are defined to coincide with the compressive field and, as with the tensile ties, begin and terminate at the nodal zones.

The truss should exhibit equilibrium at each node and should portray an acceptable truss model. The good model is should be more closely approach to the elastic stress trajectories. The poor model requires large deformation before the tie can yield, break the rule that concrete has a limited capacity to sustain plastic deformation. Figure 2.3 illustrates the difference between an acceptable model and a poor model.

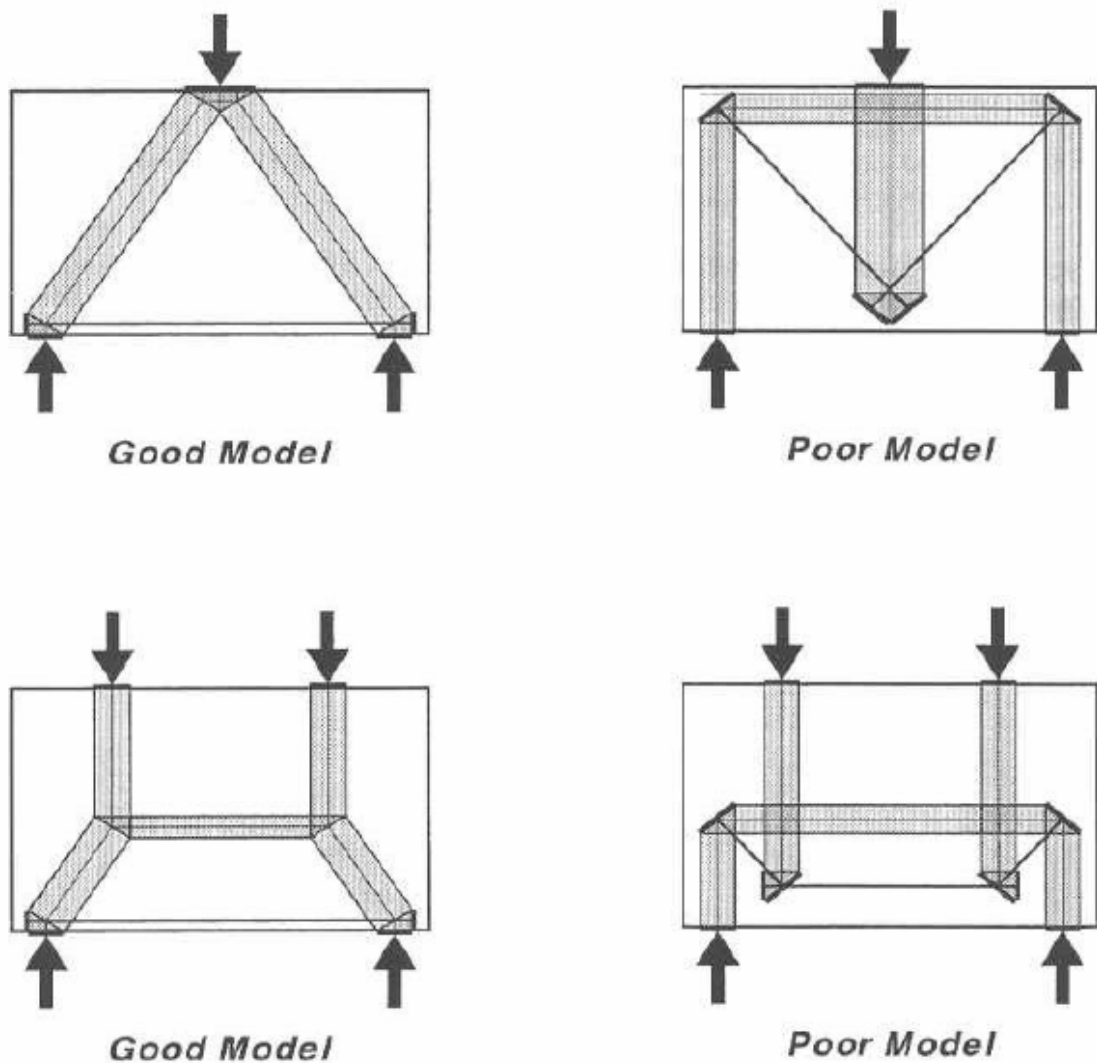


Figure 2.3 Example strut-and-tie model, An acceptable Model and Poor Model

(This figure cited from lecture note Dr.C.C. Fu, Ph.D, P.E, University of Maryland)

In a cracked structural concrete member, loads are transmitted through a set of compressive stress fields that are distributed and interconnected by a tensile stress fields. The flow of compressive stresses can be idealised using compression

members called strut, and tension stress fields are idealised using tension member called ties. Since reinforced ties are much more deformable than concrete struts, the model with the least and shortest ties should provide the most favorable model. Schlaich et al., proposes a simple criterion for optimizing a model that derived from the principle of minimum strain energy for linear elastic behavior of the struts and ties after cracking. The contribution of the concrete struts can generally be omitted because the strains of the struts are usually much smaller than those of the steel ties. An ideal arrangement of ties and strut to minimise both the forces in the various component element, and the length of the elements. This is formulated as a design criterion by as follows. Schlaich, et al [3]

$$\sum F_i l_i \varepsilon_{mi} = \text{Minimum}$$

Where

$F_i$  = force in strut or tie  $i$

$l_i$  = length of member  $i$

$\varepsilon_{mi}$  = mean strain of member  $i$

Strut-and-Tie Modeling of Structural Concrete by Dr. Quang Quan Liang et al [6], School of Civil and Environmental Engineering, The University of New South Wales, Sydney Australia developed a performance-based strut-and-tie modeling procedure for reinforced concrete citing the inefficiency of the trial-and-error iterative process that is based on the designer's intuition and past experience. Their optimization procedure consists of eliminating the most lowly stressed portions from the structural concrete member to find the actual load path. Liang, et al [6], proposes that minimizing the strain energy is equivalent to maximizing the overall stiffness of a structure and that the strut-and-tie system should be based on system performance (overall stiffness) instead of component performance (compression struts and tension ties).