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USING UPPER BOUND THEOREM**

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ANALYSIS OF THREE DIMENSIONAL GROUND STABILITY DUE TO
EXCAVATION USING UPPER BOUND THEOREM

PAN HOW YEN

A project report submitted in partial fulfillment of the
requirements for the award of the degree of
Master of Engineering (Civil - Geotechnics)

FACULTY OF CIVIL ENGINEERING
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JULY 2011

I declare that this project report entitled “*Analysis of Three Dimensional Ground Stability Due To Excavation Using Upper Bound Theorem*” is the result of my own research except as cited in the references. The report has not been accepted for any degree and is not concurrently submitted in candidature of any other degree.

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Name : PAN HOW YEN

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To my beloved mother and father

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ABSTRACT

Three-dimensional limit analysis of vertical slope is presented based on the two-dimensional approaches defined by Chen (1975). It starts with extending the two-dimension translational and rotational failure mechanisms into three-dimension along the axis that is perpendicular to the failure block in this paper. Then establishing kinematically admissible velocity field based on the new geometry of failure mechanism. Tresca's materials and Coulomb's materials are considered in the upper bound solution. A single rigid block system with finite width and energy dissipation taking place along the surface discontinuities are assumed in the three-dimension failure mechanism. The generated formulae are presented in functions of vertical slope height and stability factors. Results of the analysis are presented in graphical and table forms to show the practical range of the parameters. In practice, stability factor charts can be applied to evaluate the excavation in narrow area.

ABSTRAK

Tiga dimensi analisis untuk cerun tegak dikemukakan berdasarkan dua dimensi yang ditkrifkan oleh Chen (1975). Analisis ini bermula dengan melanjutkan mekanisme kegagalan translasi and putaran pada tiga-dimensi di sepanjang paksi yang bersudut tepat kepada paksi y. Seterusnya, medan halaju kinematik ditubuhkan berdasarkan geometri baru untuk mekanisme kegagalan. Tresca and Coulomb jenis tanah dipertimbangkan dalam penyelesaian yang terikat atas. Satu blok system dengan lebar dan kelesapan tenaga mengambil tempat di sepanjang ketidakselajaran permukaan diandaikan dalam tiga dimensi mekanisme kegagalan. Formula-formula yang dihasilkan dalam fungsi ketinggian cerun tegak and faktor-faktor kestabilan. Keputusan analisis dipersembahkan dalam bentuk grafik and jadual untuk menunjukkan praktikal rangkaian. Cakta faktor kestabilan yang dibuat boleh digunakan untuk menilai penggalan di kawasan-kawasan yang sempit.

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LIST OF SYMBOLS

σ_{ij}	-	Stress rate
δ_{ij}	-	Strain rate
F	-	body forces
T^*	-	external load
As	-	surface area
Γ	-	velocity discontinuity
V^*	-	velocity field
dD	-	rate of energy dissipation
δI	-	rate of work done by internal energy dissipation
W	-	body force from the self-weight
δ_u	-	tangential relative velocity
dV	-	volume of the single rigid block in three-dimension
τ	-	shear strength
S_u	-	undrained shear strength
δ_w	-	relative displacement
$\delta\theta$	-	relative change of rotational angle
R	-	radius
c	-	cohesion
σ_n	-	normal stress
ϕ	-	frictional angle
v_n	-	normal velocity vector
v_t	-	tangential velocity vector
u	-	pore water pressure
H	-	Height of the vertical slope
β	-	Inclined angle of failure mechanism

b	-	Width of the three dimension vertical slope
γ	-	unit weight
δ_{u_k}	-	relative tangential velocity depends on k
R_k	-	radius depends on the segments.
δE	-	rate of work done by external forces
L	-	length of failure mechanism
dz	-	rate of width
ω	-	angular velocity

CHAPTER 1

INTRODUCTION

1.1 Background of study

Slope stability analysis is one of the main concerns in geotechnical engineering. Therefore, predictions of height and limit load form an important part in slope stability analysis. In the slope stability problems, excavation works often viewed as the main earthwork activities. There are many researchers have been studying the safety margin, collapse load and critical height for the slope stability problems. Basically, slope stability problems can be distinguished into four types which are factor of safety, limit load, critical height and optimal shape of slope (Michalowski, 1989).

In order to solve the base stability problems, generally some basic conditions are needed to derive the solutions such as stress-strain relationship, stress equilibrium equations and strain-displacement equations. In soil mechanics, various method of analysis has been discussed to find the solution for the stability problems. However, they have different requirements and limitations. Limit equilibrium method was introduced by Terzaghi (1943). This method has been used extensively in solving the slope stability problems but it inherited some weaknesses such as neglects the soil stress-strain relationship (H.Y Fang, 1991). This method has some similarities with the lower bound theorem in limit analysis method. For instances, they both are satisfying equilibrium condition and force. Therefore limit equilibrium method often used to determine the stability of the slope cut by assuming the failure surface.

In 1952, Drucker and Prager started to use limit analysis method in solving slope stability problems. It consists of upper bound and lower bound theorem was then refined and extended its applications by Chen (1975). Limit analysis method was found conveniently to solve slope stability problems in different types of failure mode. Up to the present, limit analysis method often used to deal with the slope stability problems in two dimensional. Plane strain condition is applied in two dimensional by assuming infinite length that is perpendicular to the failure plane. Three dimensional failure mechanisms are less common in analysis. Previous studies of three-dimension slope stability analysis can be divided into three categories: extension of limit equilibrium method, numerical approaches and limit analysis. Upper bound theorem in limit analysis is getting more popular in the research because of the convenience and provides a close approximation to the true collapse condition.

Regarding to the slope stability, simplifications and idealizations are required to establish an appropriate solution. For instances, solution of limit analysis must satisfy soil behavior, equations of equilibrium, compatibility and boundary conditions so that real collapse condition can be predicted. The soil material is idealized as perfectly plastic and ignoring the work-hardening and work softening in the stress-strain behavior of the soil. Theoretically, satisfying equilibrium and compatibility conditions simultaneously in the work equation are difficult. Therefore, limit analysis is divided into upper bound theorem and lower bound theorem that can provide a set of bounds to the true collapse condition for the slope. According to the Figure 1.1, the dashed line is representing perfectly plastic flow stress and it was defining that the perfectly plastic flow also representing an averaging work-hardening or work-softening of the material over the field of flow by Chen (1975).

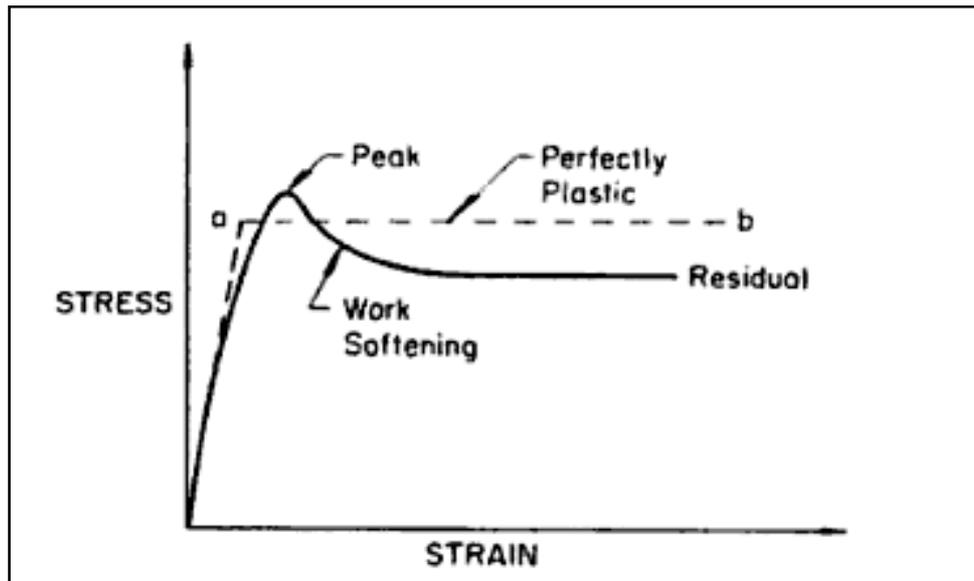


Figure 1.1: Stress-strain relationship for ideal and real soils (Chen, 1975)

Excavations are essential while constructing geotechnical structures such as foundation, slope, embankment, retaining wall, earth dam and tunneling. There are some factors affecting the ground stability due to excavations such as stress-strain relationship of the soil, failure mechanism, homogeneous or heterogeneous soil, end effects of the cut face and etc. By considering these factors, we can analyze the base stability of excavations by using limit analysis method. Hence, obtain the critical height of the excavation.

The application of upper bound theorem is able to find out the collapse load by satisfying the strain-displacement boundary condition and compatibility. In the present, this method has been successfully solved stability problems due to excavation with the aid of computational and analytical methods. In 1975, Wai-Fah Chen has refined limit analysis method and defined the failure mechanism into two types which are translational mechanism and rotational mechanism. The details were proven can be used effectively in the geotechnical design and many researches are inspired to challenge more complicated slope stability problems such as excavation in soft soil, reinforced slope stability and foundations.

With the extensive development of technologies, the relevant analyses are more reliable to apply into actual excavation stability problems. The importance of base stability due to excavation is to assure the safety during the excavation, optimize the excavation

duration and quality of work. Under these concerns, we can actually optimize the construction time and cost-effective if the results of the analysis are sufficiently accurate. In the study, upper bound theorem and valid failure mechanisms are the key to deal with the base stability of excavation.

1.2 Introduction of Upper Bound Theorem

Upper bound theorem stated that the external rate of work must be equal to the internal rate of dissipation and give an unsafe conditions or collapse must occur based on the postulated failure mechanism, at the same time satisfying the compatibility of strains and displacements. In this particular condition, the prediction of critical height is higher or equal to the actual critical height of slope. Therefore, upper bound theorem also named as unsafe analysis.

This method is adopting plasticity theory so that upper bound can be set regarding to the true collapse condition of slope. The postulated failure mechanism must have kinematically admissible velocity field and satisfying the mechanical boundary conditions. When the collapse load is achieved, the deformation of the slope remains constant and only plastic strain rates occur. Owing to the assumption of above, elastic strain rate is assumed to be small and can consider as zero for convenience. The accuracy of the upper bound solution in term of limit load, critical height and factor of safety are relying to the closeness of the true collapse condition. Apart from that, optimal failure mechanism of slope can be obtained through the comparisons between failure mechanisms including translational, rotational, circular, prism and cylindrical shapes with the same soil materials.

The general equation of upper bound theorem for any types of slope is proven by the principle of virtual work. In general, rate of work done by external forces are depending on the body forces created by self-weight and any external loads acting to the slope. The rate of work done by internal energy dissipation is depending on the cohesion developed at the surface discontinuities and frictional forces contributed by resultant normal force. For the three dimensional failure mechanisms, geometry of the failure mechanism has a better

definition than two dimensional failure mechanisms. Hence, integration can be used to define the geometry of failure mechanism in order to compute the work equation.

1.3 Problem Statement

In two dimensional failure mechanism of slope, infinite width is assumed along the axis perpendicular to the failure plane. Therefore, plain strain condition is appropriately applied in the slope stability problems. However, most of the small scale collapse region of slope has finite width. Thus, two dimensional failure mechanism of slope cannot properly model the characteristic of three dimensional failure mechanism of slope especially in landslide problems.

Besides, end effects of the three dimensional failure mechanism are important for the narrow area of excavation work such as trench excavation and stability problem related to the dam in steep-walled valleys (Baligh and Azzouz, 1975). From the theoretical point of view, three dimensional failure mechanism can be subjected to boundary constraints such as cut face and wall system which could cause the collapse mode in three dimensionally (Giger and Krizek, 1975). Previously, plain strain assumption in two dimensional failure mechanisms shows more critical than three dimensional failure mechanisms with no limit on the width of slope (Michalowski and Drescher, 2009). Therefore, well-defined width is considered in the work equations.

For the postulated failure mechanisms, upper bound theorem is used to analyze the critical height of the vertical slope. The mechanical properties are determined for the failure mechanisms. In order to satisfy the purpose of this study, some questions are aroused:

- i. What is the critical height of the unsupported vertical cut?
- ii. Which types of failure mechanism have more critical condition?
- iii. What is the practical range of width for the slope stability problems?

- iv. What is the type of soil's materials?

1.4 Objectives

The objectives of this study are stated as below:

- i. To calculate the critical height of the excavation by using upper bound theorem.
- ii. To compare the results of the different types of failure mechanism based on the same constraints.
- iii. To evaluate the practical range of parameters for the postulated failure mechanisms

1.5 Scopes

In this study, some of the limitations have been determined for the sake of the simplicity and interest in this method. Unsupported vertical slope and no tension cut are considered in the three dimensional translational and rotational failure mechanisms. Upper bound theorem is the only method adopted to derive the formulations based on the failure mechanisms which are inspired by two dimensional translational and rotational failure mechanisms (Chen, 1975).

For the sake of comparison purpose, simplifications are made such as homogeneous soil, no tension cut and ignores the pore water pressure effects. Tresca's material and Coulomb's material are considered in the formulations of this study. In general, Tresca's material is considered as undrained soil condition ($c > 0, \phi = 0$) which is incompressible and without volume change during the deformations. For instance, clayey soil subjected to undrained condition in the slope is suitable to apply with the characteristic of Tresca's material. Coulomb's material is another soil materials considered in this study. Basically, Coulomb's material is popular and often used as the first approach in the slope stability analysis. Frictional angle (ϕ) and cohesion (c) are considered in the formulations of this study. It is more complicated when compared to the Tresca's material soil condition.

Three dimensional translational and rotational failure mechanisms are the extension of two dimensional failure mechanisms by introducing parameter of width. Three dimensional failure mechanisms have better definition than two dimensional failure mechanisms in terms of geometry. Definitely, it affects the formula pattern in rate of work done by external forces and rate of work done by internal energy dissipation. The rate of work done by internal energy dissipation is no longer defined in unit area due to the better definition of geometry in failure mechanisms. For the rate of work done by external forces, width (b) is added into the formulations when computing the volume of the rigid block. Therefore dimensionless ratio of stability factor is introduced in the study which can easily obtain the critical height of the vertical slope. The formulations also derived in the function of height so that direct comparisons can be observed.

Graphs and tables are made and compared with the previous studies. Semi-graphical method is used to obtain the critical upper bound solution in the study. Therefore, practical range of the parameters can be determined and validate the formulations in this study.